4. a)

$$\frac{1}{2}+\left(\frac{6}{5}\times\frac{\cancel{3}}{\cancel{4}}\right)\times\left(\frac{2}{3}+\frac{8}{3}\right)^2-\frac{5}{12}\div\left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{6}{5}\times\left(\frac{-2+8}{3}\right)^2-\frac{5}{12}$$

$$=\frac{2}{4}+\left(-\frac{1}{4}\right)=\frac{6}{5}\times\left(\frac{6}{3}\right)^2-\frac{5}{12}$$

$$=\frac{1}{4}\qquad=\frac{6}{5}\times(2)^2-\frac{5}{12}$$

$$=\frac{24}{5}-\frac{5}{12}$$

$$=\frac{288}{60}-\frac{25}{60}$$

$$=\frac{263}{60}\text{, or } 4\frac{23}{60}$$

b)

$$\left(-\frac{5}{4}\right)\div\left(-\frac{1}{4}+\frac{3}{2}\right)\left(-\frac{1}{4}+\frac{3}{2}\right)=\left(-\frac{5}{4}\right)\div\left(-\frac{1}{4}+\frac{6}{4}\right)\left(-\frac{1}{4}+\frac{6}{4}\right)$$

$$=\left(-\frac{5}{4}\right)\div\left(-\frac{5}{4}\right)\left(-\frac{5}{4}\right)$$

$$=\left[\left(-\frac{5}{4}\right)\div\left(-\frac{5}{4}\right)\right]\left(-\frac{5}{4}\right)$$

$$=1\times\left(-\frac{5}{4}\right)$$

$$=-\frac{5}{4}$$

c)

$$\left(-\frac{7}{10}\right)\div\left(-\frac{2}{5}\right)-\left(-\frac{1}{4}\right)\times\frac{1}{2}=\left(-\frac{7}{\cancel{10}^{2}}\right)\times\left(-\frac{\cancel{5}^{1}}{2}\right)-\left(-\frac{1}{4}\right)\times\frac{1}{2}$$

$$=\frac{7}{4}-\left(-\frac{1}{8}\right)$$

$$=\frac{14}{8}-\left(-\frac{1}{8}\right)$$

$$=\frac{14}{8}+\frac{1}{8}$$

$$=\frac{15}{8}\text{, or } 1\frac{7}{8}$$

7. a)

$$\left(-\frac{2}{3}\right)\div\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}=\left(-\frac{2}{3}\right)\times\frac{4}{1}+\frac{1}{2\times2\times3}$$

$$=-\frac{8}{3}+\frac{1}{12}$$

$$=-\frac{32}{12}+\frac{1}{12}$$

$$=-\frac{31}{12}\text{, or } -2\frac{7}{12}$$

b)

$$\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}+\left(-\frac{1}{2}\right)\right]\times\frac{1}{3}=\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}+\left(-\frac{2}{4}\right)\right]\times\frac{1}{3}$$

$$=\left(-\frac{2}{3}\right)\div\left(-\frac{1}{4}\right)\times\frac{1}{3}$$

$$=\left(-\frac{2}{3}\right)\times\left(-\frac{4}{1}\right)\times\frac{1}{3}$$

$$=\frac{8}{3}\times\frac{1}{3}$$

$$=\frac{8}{9}$$

c)

$$\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}-\left(-\frac{1}{2}\right)\right]\times\frac{1}{3}=\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}+\frac{2}{4}\right]\times\frac{1}{3}$$

$$=\left(-\frac{2}{3}\right)\div\frac{3}{4}\times\frac{1}{3}$$

$$=\left(-\frac{2}{3}\right)\times\frac{4}{3}\times\frac{1}{3}$$

$$=\left(-\frac{8}{9}\right)\times\frac{1}{3}$$

$$=-\frac{8}{27}$$

d)

$$\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}+\left(-\frac{1}{2}\right)\times\frac{1}{3}\right]=\left(-\frac{2}{3}\right)\div\left[\frac{1}{4}+\left(-\frac{1}{6}\right)\right]$$

$$=\left(-\frac{2}{3}\right)\div\left[\frac{3}{12}+\left(-\frac{2}{12}\right)\right]$$

$$=\left(-\frac{2}{3}\right)\div\frac{1}{12}$$

$$=\left(-\frac{2}{\cancel{3}^{1}}\right)\times\cancel{12}^{4}$$

$$=-8$$

8. a) In the second line, the student evaluated (–2.8 + 1.5) as 1.3, when it should have been –1.3. And, in the third line, the student subtracted before dividing.

Correct solution:
(–3.7) × (–2.8 + 1.5) – 4.8 ÷ (–1.2)
= (–3.7) × (–1.3) – 4.8 ÷ (–1.2)
= 4.81 – 4.8 ÷ (–1.2)
= 4.81 – (–4)
= 8.81

b) In the second line, the student subtracted before multiplying and dividing.
Correct solution:

$$-\frac{3}{8}-\frac{\cancel{4}^{2}}{5}\times\frac{3}{\cancel{10}^{5}}\div\left(-\frac{4}{5}\right)$$
$$=-\frac{3}{8}-\frac{6}{25}\div\left(-\frac{4}{5}\right)$$
$$=-\frac{3}{8}-\frac{\cancel{6}^{3}}{25^{5}}\times\left(-\frac{5^{1}}{\cancel{4}^{2}}\right)$$
$$=-\frac{3}{8}-\left(-\frac{3}{10}\right)$$
$$=-\frac{3}{8}+\frac{3}{10}$$
$$=-\frac{15}{40}+\frac{12}{40}$$
$$=-\frac{3}{40}$$

11. a) Substitute each value in the formula $C=\frac{F-32}{1.8}$.

i) For $F = 0$,
$$C=\frac{-32}{1.8}$$
$$=17.\overline{7}$$
So, 0°F is about –18°C

ii) For $F = -40$,
$$C=\frac{-40-32}{1.8}$$
$$=\frac{-72}{1.8}$$
$$=40$$
So, –40°F is –40°C.

iii) For $F = -53$,
$$C=\frac{-53-32}{1.8}$$
$$=\frac{-85}{1.8}$$
$$=47.\overline{2}$$
So, –53°F is about –47°C.

b) Substitute each value in the formula $C=\frac{5}{9}(F-32)$.

i) For $F = 50$,
$$C=\frac{5}{9}(50-32)$$
$$=\frac{5}{\cancel{9}}\times\cancel{18}^{2}$$
$$=10$$
So, 50°F is 10°C.

ii) For $F = -13$,
$$C=\frac{5}{9}(-13-32)$$
$$=\frac{5}{^{1}\cancel{9}}\times(-\cancel{45}^{5})$$
$$=-25$$
So, –13°F is –25°C

iii) For $F = 32$,
$$C=\frac{5}{9}(32-32)$$
$$=0$$
So, 32°F is 0°C.

c) Answers may vary. For example: It seemed easier to use the formula in part a when I used a calculator. I could use mental math for most of the questions in part b.

12. a) Multiply first, then add.
$$\left(-4\frac{1}{2}\right)+\left(-\frac{2}{3}\right)\times 2\frac{3}{4}=\left(-\frac{9}{2}\right)+\left(-\frac{^{1}\cancel{2}}{3}\right)\times\frac{11}{\cancel{4}^{2}}$$
$$=\left(-\frac{9}{2}\right)+\left(-\frac{11}{6}\right)$$
$$=\left(-\frac{27}{6}\right)+\left(-\frac{11}{6}\right)$$
$$=-\frac{38}{6}, \text{ or } -6\frac{1}{3}$$

b) Multiply first, then add.
$$\left(-3\frac{2}{5}\right)\times\left(-1\frac{5}{6}\right)+\frac{3}{10}=\left(-\frac{17}{5}\right)\left(-\frac{11}{6}\right)+\frac{3}{10}$$
$$=\frac{187}{30}+\frac{3}{10}$$
$$=\frac{187}{30}+\frac{9}{30}$$
$$=\frac{196}{30}$$
$$=\frac{98}{15}, \text{ or } 6\frac{8}{15}$$

c) Divide, multiply, then add.

$$(-3)\div\left(-\frac{4}{5}\right)+\left(-\frac{5}{12}\right)\times 1\frac{1}{2}=(-3)\times\left(-\frac{5}{4}\right)+\left(-\frac{5}{12}\right)\times\frac{3}{2}$$
$$=\frac{15}{4}+\left(-\frac{5}{\cancel{12}_{4}}\right)\times\frac{\cancel{3}}{2}$$
$$=\frac{15}{4}+\left(-\frac{5}{8}\right)$$
$$=\frac{30}{8}+\left(-\frac{5}{8}\right)$$
$$=\frac{25}{8}, \text{ or } 3\frac{1}{8}$$

d) Add in brackets, multiply, then subtract.

$$\left(1\frac{5}{8}\right)-\left(-2\frac{3}{4}+2\right)\left(-2\frac{3}{4}+2\right)=\left(\frac{13}{8}\right)-\left(-\frac{11}{4}+2\right)\left(-\frac{11}{4}+2\right)$$
$$=\left(\frac{13}{8}\right)-\left(-\frac{11}{4}+\frac{8}{4}\right)\left(-\frac{11}{4}+\frac{8}{4}\right)$$
$$=\left(\frac{13}{8}\right)-\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)$$
$$=\frac{13}{8}-\frac{9}{16}$$
$$=\frac{13}{8}-\frac{9}{16}$$
$$=\frac{26}{16}-\frac{9}{16}$$
$$=\frac{17}{16}, \text{ or } 1\frac{1}{16}$$

13. a) 2.3 + (–11.2) ÷ (–0.2) – 3.7 = 54.6

b) (–3.4) × 0.7 – (–1.8)(–1.8) = –5.62

c) $\dfrac{0.67-4.2\div(-0.2)}{(-7.3+8.6)^2}\doteq 12.82$

d) $\dfrac{8.9\times(-3.1+22.7)^2+4.7}{(-9.6)\div 0.04-0.4}\doteq -14.24$

17. In the third line, the product of 2.9 × (–5.7) is negative, not positive. In the fourth line, the student subtracted before dividing.

Correct solution:

$(-8.2)^2\div(-0.3)-2.9\times(-5.7)$

$=67.24\div(-0.3)-2.9\times(-5.7)$

$=-224.1\overline{3}-2.9\times(-5.7)$

$=-224.1\overline{3}-(-16.53)$

$=-224.1\overline{3}+16.53$

$=-207.60\overline{3}$

So, the correct solution, to the nearest hundredth, is –207.60.

18. a)

$$\frac{23.7-(-5.6)\div 0.7+6.8}{(-3)\times(-6.7)+3.5}$$
$$=\frac{23.7-(-8)+6.8}{20.1+3.5}$$
$$=\frac{31.7+6.8}{23.6}$$
$$=\frac{38.5}{23.6}$$
$$\doteq 1.631$$

The correct answer, to the nearest hundredth, is 1.63.

b) The student likely calculated:

23.7 – (–5.6) ÷ 0.7 + 6.8 ÷ (–3) × (–6.7) + 3.5

≐ 23.7 + 5.6 ÷ 0.7 + 15.186 + 3.5

= 23.7 + 8 + 18.686

= 50.386

instead of calculating the numerator and the denominator, then finding the result of the division:

[23.7 – (–5.6) ÷ 0.7 + 6.8] ÷ [(–3) × (–6.7) + 3.5]

19. Multiplying by $\frac{5}{9}$ is equivalent to dividing by $\frac{9}{5}$, or 1.8.

To get from $C=\frac{5}{9}(F-32)$ to $C=\frac{F-32}{1.8}$:

$$C=\frac{5}{9}(F-32)$$
$$C=\frac{5}{9}\times(F-32)$$
$$C=(F-32)\times\frac{5}{9}$$
$$C=(F-32)\div\frac{9}{5}$$
$$C=(F-32)\div 1.8$$
$$C=\frac{F-32}{1.8}$$

To get from $C=\frac{F-32}{1.8}$ to $C=\frac{5}{9}(F-32)$:

$$C=\frac{F-32}{1.8}$$
$$C=(F-32)\div 1.8$$
$$C=(F-32)\div\frac{18}{10}$$

$C = (F - 32) \times \frac{\cancel{10}^{5}}{{}^{9}\cancel{18}}$

$C = \frac{5}{9}(F - 32)$

20. Let $\square$ represent the minimum temperature.

$\frac{\square + (-11.5)}{2} = -12.8$

$\square + (-11.5) = -25.6$

$\square = (-25.6) - (-11.5)$

$\square = -14.1$

The minimum temperature was –14.1°C.

21. I used guess and test.

$-3.8 + 9.1 \times (-2.5 - 0.5) = -31.1$

To get a positive answer, I inserted the brackets so that I ended up having a product of two factors with the same sign.

$-[3.8 + 9.1 \times (-2.5) - 0.5]$

$= (-1)[3.8 - 22.75 - 0.5]$

$= (-1) \times (-19.45)$

$= 19.45$